THE UNIVERSITY OF POONCH RAWALAKOT

AZAD JAMMU AND KASHMIR

Scheme of study for M. Sc.(2-Years) program in Mathematics

Duration of program:	4-6 semesters
Courses:	60 credits
Conference / Seminar / Reading	01 credits
Research Thesis (optional):	6 credits
Total credits:	61 credits
Comprehensive Oral examination:	Satisfactory/Unsatisfactory basis

Semester-I

Course Code	Course Title	Credit hours
MAT-5101	Real Analysis-I	3(3-0)
MAT-5102	Differential Geometry-I	3(3-0)
MAT-5103	Algebra-I	3(3-0)
MAT-5104	Topology-I	3(3-0)
MAT-5105	Ordinary Differential Equations	3(3-0)
	Total	**15**

Semester-II

Course Code	Course Title	Credit hours
MAT-5201	Real Analysis-II	3(3-0)
MAT-5202	Complex Analysis	3(3-0)
MAT-5203	Algebra-II	3(3-0)
MAT-5204	Functional Analysis-I	3(3-0)
MAT-5205	Classical Mechanics	3(3-0)
	Total	**15**

Semester-III

Note: Conference / Seminar / Reading & any five courses from the following elective courses.

Course Code	Course Title	Credit hours
MAT-6301	Numerical Analysis-I	3(3-0)
MAT-6302	Partial Differential Equations	3(3-0)
MAT-6303	Functional Analysis-II	3(3-0)
MAT-6304	Fluid Mechanics-I	3(3-0)
MAT-6305	Measure Theory	3(3-0)
MAT-6306	Modeling and Simulation	3(2-1)
MAT-6307	Optimization Theory	3(3-0)
MAT-6308	Mathematical Statistics-I	3(3-0)
MAT-6309	Theory of Differential Equations	3(3-0)
MAT-6310	Analysis and Control of Nonlinear Systems	3(3-0)
MAT-6311	Numerical Methods for Partial-Differential Equations	3(3-0)
MAT-6312	Ring Theory	3(3-0)
MAT-6313	Vector and Tensor Analysis	3(3-0)
MAT-6314	Computing Tools for Mathematicians	3(2-1)
MAT-6315	Number Theory	3(3-0)
MAT-6399	Conference / Seminar / Reading	1(1-0)
	Total	**16**

Semester-VI

Note: Any five courses or three courses and thesis from the following elective courses.

Course Code	Course Title	Credit hours
MAT-6401	Numerical Analysis-II	3(3-0)
MAT-6402	Differential Geometry-II	3(3-0)
MAT-6403	Mathematical Physics	3(3-0)
MAT-6404	Fluid Mechanics-II	3(3-0)
MAT-6405	Quantum Mechanics	3(3-0)
MAT-6406	Integral Equations	3(3-0)
MAT-6407	An Introduction to Convex Analysis	3(3-0)
MAT-6408	Calculus of Variation and Optimal Control	3(3-0)
MAT-6409	Dynamical Systems	3(3-0)
MAT-6410	Computer Language C/C++	3(2-1)
MAT-6411	Electromagnetism	3(3-0)
MAT-6412	Special Theory of Relativity	3(3-0)
MAT-6413	Topology-II	3(3-0)
MAT-6414	Theory of Elasticity	3(3-0)
MAT-6415	Decomposition of Modules	3(3-0)
MAT-6416	Mathematical Statistics-II	3(3-0)
MAT-6417	Thesis	6(6-0)
	Total	**15**

Course Contents for M. Sc. (2-Years) Program

The contents for the courses of Mathematics for the said program are presented for discussion.

Courses for Semester-I

MAT-5101 Real Analysis-I 3(3-0)

Prerequisite(s): Calculus-III

Specific Objectives of the Course:

This is the first rigorous course in analysis and has a theoretical emphasis. It rigorously develops the fundamental ideas of calculus and is aimed to develop the student's ability to deal with abstract mathematics and mathematical proofs.

Course Outline:

Ordered sets, supremum and infimum, completeness properties of the real numbers, limits of numerical sequences; limits and continuity, properties of continuous functions on closed bounded intervals; derivatives in one variable; the mean value theorem; Sequences of functions, power series, point-wise and uniform convergence. Functions of several variables: open and closed sets and convergence of sequences in R^n; limits and continuity in several variables, properties of continuous functions on compact sets; differentiation in n-space; the Taylor series in R^n with applications; the inverse and implicit function theorems.

Recommended Books:

[1] Bartle RG, Sherbert DR, Introduction to Real Analysis (3rd edition), 1999, John Wiley, New York.

[2] Brabenec RL, Introduction to Real Analysis, 1997, PWS Publishing Company.

[3] Gaughan ED, Introduction to Analysis (5th edition), 1997, Brooks/Cole.

[4] Rudin W, Principles of Mathematical Analysis (3rd edition), 1976, McGraw Hill, New York.

MAT-5102 Differential Geometry-I 3(3-0)

Prerequisite(s): Calculus-III, Linear Algebra

Specific Objectives of the Course:

The aim of the module is to describe how techniques from advanced calculus and linear algebra may be used to give meaning to the concept of "shape" for curves and surfaces in space.

Course Outline:

Historical background; motivation and application; index notation and summation convention; space curves; the tangent vector field; reparametrization; arc length; curvature; principal normal; binormal; torsion; osculating, normal and the rectifying planes; Frenet-Serret Theorem; spherical images; sphere curves; spherical contacts; fundamental theorem of space curves; line integrals and Green's Theorem; local surface theory; coordinate transformations; the tangent and the normal planes; parametric curves; the first fundamental form and the metric tensor; normal and geodesic curvatures; Gauss's formula Christoffel symbols of first and second kinds; parallel vector fields along a curve and parallelism; second fundamental form and the Weingarten map; principal, Gaussian, mean and normal curvatures.

Recommended Books:

[1] Millman RS, Parker GD, Elements of Differential Geometry, 1977, Prentice-Hall Inc, New Jersey.

[2] Struik DJ, Lectures on Classical Differential Geometry, 1977, Addison-Wesley Publishing Company, Inc, Massachusetts.

[3] Do Carmo MP, Differential Geometry of Curves and Surfaces, 1985, Prentice-Hall Inc, Englewood New Jersey.

[4] Goetz A, Introduction to Differential Geometry, 1970, Addison-Wesley.

MAT-5103 **Algebra-I** **3(3-0)**

Prerequisite(s): Linear Algebra

Specific Objectives of the Course:

This is the first course in group theory, which provides basic background needed for all mathematics majors, a prerequisite for many courses. Many concepts presented in the course are based on the familiar setting of set theory, and are developed with an awareness of how abstract algebra is applied.

Course Outline:

Groups and subgroups; generators and relations; cyclic groups; cosets and Lagrange's theorem; normalizers and centralizers; centre of a group; conjugacy classes of groups; normal subgroups and simple groups; factor groups; isomorphism theorems and automorphisms; commutators; permutation groups and Cayley's theorem; symmetric groups; finite p-groups, internal and external direct products, group action on sets, isotropy subgroups, orbits, 1^{st}, 2^{nd} and 3^{rd} Sylow theorems.

Recommended Books:

[1] MacDonald ID, The Theory of Groups, 1968, Oxford University Press.

[2] Fraleigh J B, A First Course in Algebra, 1982, Addison-Wesley.

[3] Hamermesh M, Group Theory and its Application to Physical Problems, 1982, Addison Wesley.

[4] Herstein IN, Topics in Algebra, 2004, John-Willey.

[5] Allenby RBJT, *Rings, Fields and Groups: An Introduction to Abstract Algebra*, 1983, Edward Arnold.

MAT-5104 **Topology-I** **3(3-0)**

Prerequisite(s): Real Analysis-I

Specific Objectives of the Course:

A topological space is a generalization of a metric space (which is a set with a distance function). In this course, we will review the relevant properties of metric spaces, and then study important properties of topological spaces, such as bases, connectedness, compactness, continuous functions, product spaces, and quotient spaces.

Course Outline:

Topological spaces; bases and sub-bases; subspace topology; interior, exterior and limit points; neighborhood; product spaces; continuous mappings and homeomorphism; topological properties; metric and topology induced by metric; metrizable spaces; connectedness; connected subspaces of R; open cover; compact spaces and their characterization; compact subspaces of R and R^n; first and second axioms of countability; separation axioms; regular and normal spaces; continuity in metric spaces; Cauchy sequence and complete metric space; uniform continuity; Cantor's Intersection theorem; Baire's category theorem.

Recommended Books:

[1] Simmon GF, *Introduction to Topology and Modern Analysis*, 1963, McGraw Hill, New York.

[2] Majeed A, Elements of Topology and Functional Analysis, 2000, Ilmi Kitab Khana, Lahore.

[3] Munkres JR, *Topology: A First Course*, Prentice Hall, Englewood Cliffs, NJ, USA.

MAT-5105 Ordinary Differential Equations 3(3-0)

Prerequisite(s): Calculus-III

Specific Objectives of the Course:

This course provides the foundation of all advanced subjects in Mathematics. Strong foundation and applications of Ordinary Differential Equations is the goal of the course.

Course Outline:

Introduction; formation, solution and applications of first-order-differential equations; formation and solution of higher-order-linear-differential equations; differential equations with variable coefficients; Sturm-Liouville (S-L) system and boundary-value problems; series solution and its limitations; the Frobenius method, solution of the Bessel, the hypergeometric, the Legendre and the Hermite equations, properties of the Bessel function.

Recommended Books:

[1] Zill DG, Cullen MR, Differential Equations with Boundary-Value Problems, (3rd Edition), 1997, PWS Publishing Co.

Courses for Semester-II

MAT-5201 Real Analysis-II 3(3-0)

Prerequisite(s): Real Analysis-I

Specific Objectives of the Course:

A continuation of Real Analysis I, this course rigorously develops integration theory. Like Real Analysis-I, Real Analysis-II emphasizes proofs.

Course Outline:

Series of numbers and their convergence; Series of functions and their convergence; Dabroux upper and lower sums and integrals; Dabroux integrability; Riemann sums and the Riemann integral; Riemann integration in R^2; change of order of variables of integration; Riemann integration in R^3 and R^n; Riemann-Steiltjes integration; Functions of bounded variation; The length of a curve in R^n.

Recommended Books:

[1] Bartle RG, Sherbert DR, Introduction to Real Analysis (3rd edition), 1999, John Wiley, New York.

[2] Brabenec RL, *Introduction to Real Analysis*, 1997, PWS Publishing Company.

[3] Fulks W, *Advanced Calculus*, John Wiley, New York (suggested text).

[4] Gaughan ED, *Introduction to Analysis* (5th edition), 1997, Brooks/Cole.

[5] Rudin W, *Principles of Mathematical Analysis* (3rd edition), 1976, McGraw Hill, New York.

MAT-5202 Complex Analysis 3(3-0)

Prerequisite(s): Real Analysis-I

Specific Objectives of the Course:

This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis and especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context.

Course Outline:

The algebra and the geometry of complex numbers, Cauchy-Riemann equations, harmonic functions, elementary functions, branches of the logarithm, complex exponents. Contours and contour integrals, the Cauchy-Goursat Theorem, Cauchy integral formulas, the Morera Theorem, maximum modulus principle, the Liouville theorem, fundamental theorem of algebra.

Convergence of sequences and series, the Taylor series, the Laurent series, uniqueness of representation, zeros of analytic functions. Residues and poles and the residue theorem, evaluation of improper integrals involving trigonometric functions, integrals around a branch point., the argument principle, the Roche theorem.

Recommended Books:

[1] Churchill RV, Brown JW, Complex Variables and Applications (5th edition), 1989, McGraw Hill, New York.

MAT-5203 Algebra-II 3(3-0)

Prerequisite(s): Introduction to Linear Algebra

Specific Objectives of the Course:

This is a course in advanced linear algebra, which builds on the concepts learnt in Linear Algebra.

Course Outline:

Review of elementary concepts of vector spaces, Linear dependence and independence of vectors. Vector spaces and subspaces, Quotient Spaces, Direct sum of spaces, Linear transformation, Rank and Nullity of linear transformations, Algebra of linear transformation and representation of linear transformation of matrices, Change of bases, Linear functionals, Dual spaces and Annihilators, Eigenvectors and eigenvalues and Cayley-Hamilton Theorem, Diagonalization of matrices, Inner product spaces, Matrices for point group operations.

Recommended Books:

[1] Shilov GE, Linear Algebra, 1997, Dover Publications, Inc. New York.

[2] Zill DG, Cullen MR, Advanced Engineering Mathematics, 1996, PWS Publishing Company Baston.

[3] Herstein IN, Topics in Algebra, 1975, John-Wiley.

[4] Trooper AM, Linear Algebra, 1969, Thomas Nelson and Sons.

[5] Lang S, Linear Algebra, 2005, Thomas Nelson and Sons.

[6] Scheick JT, Linear Algebra with Applications, 1997, McGraw Hill.

MAT-5204 Functional Analysis-I 3(3-0)

Prerequisite(s): Complex Analysis

Specific Objectives of the Course:

This course extends methods of linear algebra and analysis to spaces of functions, in which the interaction between algebra and analysis allows powerful methods to be developed. The course will be mathematically sophisticated and will use ideas both from linear algebra and analysis.

Course Outline:

Normed Spaces: Linear spaces, Normed spaces, Difference between a metric and a normed space, Banach spaces, Bounded and continuous linear operators and functionals, Dual spaces, Finite dimensional spaces, F. Riesz Lemma, The Hahn-Banach Theorem, The HB theorem for complex spaces, The HB theorem for normed spaces, The open mapping theorem, The closed graph theorem, Uniform boundedness principle and its applications.

Banach-Fixed-Point Theorem: Applications in Differential and Integral equations.

Inner-Product Spaces: Inner-product space, Hilbert space, orthogonal and orthonormal sets, orthogonal complements, Gram-Schmidt orthogonalization process, representation of functionals, Reiz-representation theorem, weak and weak* Convergence.

Recommended Books:

[1] Curtain RF, Pritchard AJ, Functional Analysis in Modern Applied Mathematics, Aademic Press, New York.

[2] Friedman A, *Foundations of Modern Analysis*, 1982, Dover.

[3] Kreyszig E, Introductory Functional Analysis with Applications, John Wiley, New York.

[4] Rudin W, *Functional Analysis*, 1973, McGraw Hill, New York.

MAT-5205 Classical Mechanics 3(3-0)

Prerequisite(s): Vector and Tensor Analysis

Specific Objectives of the Course:

This course builds grounding in principles of classical mechanics, which are to be used while studying quantum mechanics, statistical mechanics, electromagnetism, fluid dynamics, space-flight dynamics, astrodynamics and continuum mechanics.

Course Outline:

Particle kinematics, radial and transverse components of velocity and acceleration, circular motion, motion with a uniform acceleration, the Newton laws of motion (the inertial law, the force law and the reaction law), newtonian mechanics, the newtonian model of gravitation, simple-harmonic motion, damped oscillations, conservative and dissipative systems, driven oscillations, nonlinear oscillations, calculus of variations, Hamilton’s principle, lagrangian and hamiltonian dynamics, symmetry and conservation laws, Noether’s theorem, central-force motion, two-body problem, orbit theory, Kepler’s laws of motion (the law of ellipses, the law of equal areas, the harmonic law), satellite motion, geostationary and polar satellites, kinematics of two-particle collisions, motion in non-inertial reference frame, rigid-body dynamics (3-D-rigid bodies and mechanical equivalence, motion of a rigid body, inverted pendulum and stability, gyroscope).

Recommended Books:

[1] Bedford A, Fowler W, Dynamics: Engineering Mechanics, Addision-Wesley, Reading, Ma, USA.

[2] Chow TL, Classical Mechanics, 1995, John Wiley, New York.

[3] Goldstein H, Classical Mechanics (2^{nd} edition), 1980, Addison-Wesley, Reading,Ma, USA.

[4] Marion JB, Classical Dynamics of Particles and Fields (2^{nd} edition), 1970, Academic Press, New York (suggested text).

Courses for Semester-III

MAT-6301 Numerical Analysis-I 3(3-0)

Prerequisite(s): Computing Tools for Mathematicians

Specific Objectives of the Course:

This course is designed to teach the students about numerical methods and their theoretical bases. The students are expected to know computer programming to be able to write program for each numerical method. Knowledge of calculus and linear algebra would help in learning these methods.

Course Outline:

Computer arithmetic, approximations and errors; methods for the solution of nonlinear equations and their convergence: bisection method, regula falsi method, fixed point iteration method, Newton-Raphson method, secant method; error analysis for iterative methods. Interpolation and polynomial approximation: Lagrange interpolation, Newton’s divided difference, forward-difference and backward-difference formulae, Hermite inter-polation. Numerical integration and error estimates: rectangular rule, trapezoidal rule, Simpson’s one-three and three-eight rules. Numerical solution of systems of algebraic linear equations: Gauss-elimination method, Gauss-

Jordan method; matrix inversion; LU-factorization; Doolittle's, Crount's, Cholesky's methods; Gauss-Seidel and Jacobi methods. **Recommended Books:**

[1] Atkinson KE, An Introduction to Numerical Analysis (2nd edition), 1989, John Wiley, New York (suggested text).

[2] Burden RL, Faires JD, *Numerical Analysis* (5th edition), 1993, PWS Publishing Company.

[3] Chapra SC, Canale RP, *Numerical Methods for Engineers*, 1988, McGraw Hill, New York.

MAT-6302 Partial Differential Equations 3(3-0)

Prerequisite(s): Ordinary Differential Equations, Real analysis

Specific Objectives of the Course:

The course provides a foundation to solve Partial Differential Equations with special emphasis on wave, heat and Laplace equations. Formulation and some theory of these equations are also intended.

Course Outline:

First-order-partial-differential equations; classification of second-order PDE; canonical form for second-order equations; wave, heat and the Laplace equation in Cartesian, cylindrical and spherical-polar coördinates; solution of partial differential equation by the methods of: separation of variables; the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations.

Recommended Books:

[1] Myint UT, Partial Differential Equations for Scientists and Engineers (3rd edition), 1987, North Holland, Amsterdam.

[2] Sneddon lN, Elements of partial Differential Equation, 1987, McGraw-Hill Books Company.

[3] Dennemyer R, Introduction to Partial Differential Equations and Boundary Value Problems, 1968, McGraw-Hill Book Company.

[4] Humi M, Miller WB, Boundary Value Problems and Partial Differential Equations, 1992, PWS-Kent Publishing Company, Bostan.

[5] Chester CR, Techniques in Partial Differential Equations, 1971, McGraw-Hill Book Company.

MAT-6303 Functional Analysis-II 3(3-0)

Prerequisite(s): Functional Analysis-I

Specific Objectives of the Course:

Upon successful completion of this course students will be able to familiar with the major concepts of modern functional analysis, which certainly increase their level of mathematical maturity, including discovering and writing their own proofs.

Course Outline:

The Hahn-Banach theorem, Principle of uniform boundedness, Open mapping theorem, Closed graph theorem, Weak topologies and the Banach-Alaoglu theorem, Extreme points and the Klien-Milman theorem, The dual and bidual spaces, Reflexive spaces, Compact operators, Spectrum and eigenvalues of an operator, Elementary spectral theory.

Recommended Books:

[1] Kreyszing E, Introductory Functional Analysis and Applications, 1973, John Wiley.

[2] Taylor E, Lay DC, Introduction of Functional Analysis, 1979, John Wiley.

[3] Heuser HG, Functional Analysis, 1982, John Wiley.

[4] Groetsch CW, Elements of Applicable of Applicable Functional Analysis, 1980, Marcel Dekker.

MAT-6304 **Fluid Mechanics-I** **3(3-0)**

Prerequisite(s): Classical Mechanics

Specific Objectives of the Course:

The course objective is to provide students with the fundamental physical and analytical principles of fluid mechanics through the understanding of the: conservation of mass, conservation of energy, and the conservation of momentum equations. The student will demonstrate the understanding of these fundamentals by solving problems dealing with: fluid properties, fluid statics, pressure on plane and curved surfaces, buoyancy and floatation, kinematics, systems, control volumes, conservation principles, ideal incompressible flow, impulse-momentum, and flow of a real fluid.

Course Outline:

Real fluids and ideal fluids, Velocity of a fluid at a point, Equation of continuity, Acceleration of a fluid conditions at a rigid boundary, general analysis of fluid motion, Streamlines and path lines, Steady and unsteady flows, Velocity potential, Vorticity vector, Local and particle rates of change, Boundary Conditions, Boundary surface and its applications. Euler’s equations of motion, Bernoulli’s equation Steady motion under conservative body forces, Some potential theorems. Sources sink and doublets, Images in rigid infinite plane and solid spheres, A-symmetric flows, Stokes’s stream function. Stream function, Complex potential for two-dimensional, Irrotational, Incompressible flow, Complex velocity potential for uniform stream, Line Sources and line sinks, Line doublets and line vortices, Image system, Milne-Thomson circle theorem, Blasius’s theorem, The use of conformal transformation and the Schwarz-Christoffel transformation in solving problems, Vortex rows, Kelvin’s Minimum energy theorem, Uniqueness theorem, Fluids streaming past a circular cylinder, Irrotational motion produced by a vortex filament, Karman’s vortex-street.

Recommended Books:

[1] Charlton F, Textbook of fluid Dynamics, 1967, D. Van Nostrand Co. Ltd.

[2] Thomson M, Theoretical Hydrodynamics, 1979, Macmillan Press.

[3] Jaunzemics W, Continuum Mechanics, 1967, Macmillan Company.

[4] Landu LD, Lifshitz EM, Fluid Mechanics, 1966, Pergamon Press.

[5] Batchelor GK, An Introduction to Fluids Dynamics, 1969, Cambridge University Press.

MAT-6305 **Measure Theory** **3(3-0)**

Prerequisite(s): Real Analysis-I

Specific Objectives of the Course:

To introduce the concepts of *measure* and *integral with respect to a measure,* to show their basic properties, and to provide a basis for further studies in Analysis, Probability, and Dynamical Systems. Moreover, to gain understanding of the abstract measure theory and definition and main properties of the integral. To construct Lebesgue's measure on the real line and in n-dimensional Euclidean space. To explain the basic advanced directions of the theory.

Course Outline:

Measure Space; Definition and examples of algebra and σ-algebra, Basic properties of measurable spaces, Definition and examples of measure spaces, Outer measure, Lebesgue measure, Measurable sets, complete measure spaces.

Measurable Functions; Some equivalent formulations of measurable functions, Examples of measurable functions, Various characterization of measurable functions, Properties that holds almost everywhere.

Lebesgue Integration; Definition of Lebesgue integral, Basic properties of Lebesgue integrals, Comparison between Riemann integration and Lebesgue integration, L_2-Spaces, The Riesz Fischer theorem.

Recommended Books:

[1] Royden HL, Real Analysis, 1968, Mecmillian.

[2] Cohn DL, Measure Theory, 1980, Birkhauser.

[3] Halmos PR, Measure Theory, 1950, D. Van Nostrand.

[4] Khan AR, Introduction to Lebesgue Integration, 2003, Ilmi Kitab Khanna, Urdu Bazar, Lahore.

MAT-6306 Modeling and Simulation 3(2-1)

Prerequisite(s): Partial Differential Equations

Specific Objectives of the Course:

Mathematics is used in many areas such as engineering, ecological systems, biological systems, financial systems, economics, etc. In all such applications one approximates the actual situation by an idealized model. This is an introductory course of modeling, consisting of three parts: modeling with ordinary differential equations and their systems; partial differential equations; and integral equations. The course will not be concerned with the techniques for solving the equations but with setting up the equations in specific applications. Whereas the first two types of equations have already been dealt with, the third type has not. Consequently, solutions of the former will be discussed but of the latter will barely be touched upon.

Course Outline:

Concepts of model, modeling and simulation, functions, linear equations, linear-differential equations, nonlinear-differential equations and integral equations as models, introduction to simulation techniques.

Ordinary-Differential Equations: Modeling with first order differential equations: Newton's law of cooling; radioactive decay; motion in a gravitational field; population growth; mixing problem; Newtonian mechanics. Modeling with second order differential equations: vibrations; application to biological systems; modeling with periodic or impulse forcing functions. Modeling with systems of first order differential equations; competitive hunter model; predator-prey model.

Partial-Differential Equations: Methodology of mathematical modeling; objective, background, approximation and idealization, model validation, compounding. Modeling wave phenomena (wave equation); shallow water waves, uniform transmission line, traffic flow, RC circuits. Modeling the heat equation and some application to heat conduction problems in rods, lamina, cylinders etc. Modeling the potential equation (Laplace equation), applications in fluid mechanics, gravitational problems. Equation of continuity.

Simulation: Techniques of simulation (students are required to simulate at least one system).
Recommended Books:

[1] Giordano FR, Weir MD, Differential Equations: A Modeling Approach, 1994, Addison-Wesley, Reading, Ma, USA (suggested text).

[2] Jerri AJ, Introduction to Integral Equations with Applications, 1985, Marcel Dekker, New York.

[3] Myint UT, Debnath L, Partial Differential Equations for Scientists and Engineers (3rd edition), 1987, North Holland, Amsterdam.

MAT-6307 **Optimization Theory** **3(3-0)**

Prerequisite(s): Real Analysis-I

Specific Objectives of the Course:

The main objective is to teach the basic notions and results of mathematical programming and optimization. The focus will be to understand the concept of optimality conditions and the construction of solutions. Students should have a good background in analysis, linear algebra and differential equations.

Course Outline:

Linear programming: simplex method, duality theory, dual and primal-dual simplex methods. Unconstrained optimization: optimality conditions, one-dimensional problems, multi-dimensional problems and the method of steepest descent. Constrained optimization with equality constraints: optimality conditions, Lagrange multipliers, Hessians and bordered Hessians. Inequality constraints and the Kuhn-Tucker Theorem. The calculus of variations, the Euler-Lagrange equations, functionals depending on several variables, variational problems in parametric form, transportation models and networks.

Recommended Books:

[1] Elsgolts L, Differential Equations and the Calculus of Variations, 1970, Mir Publishers, Moscow.

[2] Gotfried BS, Weisman J, Introduction to Optimization Theory, 1973, Prentice Hall, Englewood Cliffs, NJ, USA.

[3] Luenberger DG, Introduction to Linear and Non-Linear Programming, 1973, Addision-Wesley, Reading, Ma, USA.

MAT-6308 **Mathematical Statistics-I** **3(3-0)**

Prerequisite(s): Calculus III

Specific Objectives of the Course:

This course is designed to teach the students how to handle data numerically and graphically. If data are influenced by *chance* effect, the concepts and rules of probability theory may be employed, being the theoretical counterpart of the observable reality, whenever *chance* is at work.

Course Outline:

Introduction to probability theory; random variables; probability distributions; mean, standard deviation, variance and expectation, Binomial, negative binomial, Poisson, geometric, hypergeometric and normal distributions; normal approximation to binomial distribution; distributions of 2 random variables.

Recommended Books:

[1] DeGroot MH, Schervish MJ, *Probability and Statistics* (3rd edition), 2002, Addison-Wesley, Reading, Ma, USA (suggested text).

[2] Papoulis A, *Probability, Random Variables, and Stochastic Processes*, (3rd edition), 1991, McGraw Hill, New York.

[3] Sincich T, *Statistics by Examples*, 1990, Dellen Publishing Company.

[4] Sincich T, *Statistics by Examples*, 1990, Dellen Publication Company.

MAT-6309 Theory of Differential Equations 3(3-0)

Prerequisite(s): Ordinary Differential Equations

Specific Objectives of the Course:

The aim of this course is to introduce the students to the basic theory of ordinary differential equations and give a competence in solving ordinary differential equations by using analytical or numerical methods. The subject of differential equations is a very important branch of applied mathematics. Many phenomena from physics, biology and engineering may be described using ordinary differential equations. In order to understand the underlying processes we have to find and interpretate the solutions of these equations. This course explains different methods of solution of ordinary differential equations: from analytical to numerical.

Course Outline:

First-Order Differential Equations; Linear Systems; Autonomous Systems; Perturbation Methods; The Self-Adjoint Second-Order Differential Equation; Linear Differential Equations of Order n; BVPs for Nonlinear Second-Order DEs; Existence and Uniqueness Theorems.

Recommended Books:

[1] Hartman P, Ordinary Differential Equations, 1964, John Wiley, New York.

[2] Hubbard JH, West BH, Differential Equations: a Dynamical Systems Approach, Higher-Dimensional Systems, 1995, Springer-Verlag, New York.

[3] Iooss G, Joseph DD, Elementary Stability and Bifurcation Theory, Undergraduate Texts in Mathematics, 1980, Springer-Verlag, New York.

[4] Walter GK, Allan CP, The Theory of Differential Equations: Classical & Qualitative, 2010, Springer, New York.

MAT-6310 Analysis and Control of Nonlinear Systems 3(3-0)

Prerequisite(s): Ordinary Differential Equations

Specific Objectives of the Course:

The course aims to provide an overview of techniques for analysis and control design for nonlinear systems. Whereas linear system control theory is largely based on linear algebra (e.g. when determining the behavior of solutions to linear differential equations) and complex analysis (e.g. when predicting system behavior from transfer functions), the broader range of behavior exhibited by nonlinear systems requires a wider variety of techniques. The course gives an introduction to some of the most useful and commonly used tools for determining system behavior from a description in terms of differential equations.

Course Outline:

Introduction to Nonlinear Phenomena: Multiple Equilibria, Limit Cycles, Complex Dynamics, Bifurcations. Second Order Nonlinear Systems: Phase Plane Techniques, Limit Cycles - Poincare-Bendixson Theory, Index Theory. Input-output analysis and stability: Small Gain Theorem, Passivity, Describing Functions. Lyapunov Stability Theory: Basic stability and instability theorems, LaSalle’s theorem, Indirect method of Lyapunov. Linearization by State Feedback: Input-Output and Full State Linearization, Zero Dynamics, Inversion, Tracking, Stabilization.

Recommended Books:

[1] Sastry SS, Nonlinear Systems: Analysis, Stability, and Control, 1999, Springer-Verlag.

[2] Khalil HK, Nonlinear Systems, 2002, Prentice-Hall.

MAT-6311 Numerical Methods for Partial Differential Equations 3(3-0)

Prerequisite(s): Partial Differential Equations

Specific Objectives of the Course:

Many application problems lead to partial differential equations in which analytic solutions are rarely available or too complicated. Finite difference methods are often easy to use but powerful to obtain an approximate solution of the PDE. It is strongly believed that the knowledge of *Finite Difference Methods for PDEs* is important for mathematicians, scientists, and engineers, who are interested in solving their problems approximately. It is a required course in many universities. This course is designed for students in applied mathematics, engineering, and the sciences to learn the basic *theories and algorithms* of finite difference methods for differential equations including elliptic, parabolic and hyperbolic PDE's. While theoretical foundations will be described, emphasis will also be placed on algorithm design and implementation. We will also explore available software packages in this field.

Course Outline:

Introduction (ODE/PDE and classification, analytic approaches versus numerical approximation, finite difference versus finite element method); A model problem (two point boundary value problem) and the finite difference method; Finite difference methods basics, stability and consistency, etc; Finite difference method for general one dimensional elliptic boundary value problems with different boundary conditions; Finite difference methods for two dimensional elliptic PDEs (may include multi-grid and fast Poisson solvers); Finite difference methods for one and two dimensional parabolic PDEs, e.g. the heat equation; von Neumann stability analysis and Fourier transforms, ADI method; Finite difference methods for one and two dimensional hyperbolic PDEs, e.g. the wave equation, numerical methods for conservation laws; Advanced topics, irregular domain, the level set method etc. if time permits.

Recommended Books:

[1] LeVeque RJ, Finite Difference Methods for Ordinary and Partial Differential Equations, 2007, SIAM.

[2] Strilwerda JC, Finite Difference Schemes and Partial Differential Equations, 1989, Chapman & Hall.

[3] Morton KW, Mayers DF, Numerical Solution of Partial Differential Equations, 1995, Cambridge press.

MAT-6312 Ring Theory 3(3-0)

Prerequisite(s): Algebra-I

Specific Objectives of the Course:

Upon successful completion of this course students will be able to: demonstrate knowledge of the syllabus material; write precise and accurate mathematical definitions of objects in ring theory; use mathematical definitions to identify and construct examples and to distinguish examples from non-examples; validate and critically assess a mathematical proof; use a combination of theoretical knowledge and independent mathematical thinking to investigate questions in ring theory and to construct proofs; and write about ring theory in a coherent, grammatically correct and technically accurate manner.

Course Outline:

Definitions and basic concepts, Homomorphisms, Homomorphism theorems, Polynomial rings, Unique factorization domain, Factorization theory, Euclidean domains, Arithmetic in Euclidean domains, Extension, Algebraic and Transcendental elements, Simple extension, Introduction to Galois theory.

Recommended Books:

[1] Fraleigh JA, A first course in Abstract Algebra, 1982, Addison-Wesley publishing Company.

[2] Herstein IN, Topics in Algebra, 1975, John Wiley & Sons.

[3] Lang S, *Algebra*, 1975, Addison–Wesley.

[4] Hartley B, Hawkes TO, Rings, Modules and Linear Algebra, 1980, Chapman and Hall.

MAT-6313 Vector and Tensor Analysis 3(3-0)

Prerequisite(s): Calculus-II

Specific Objectives of the Course:

This course shall assume background in calculus. It covers basic principles of vector analysis, which are used in mechanics.

Course Outline:

3-D vectors, summation convention, kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance and applications, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coördinate systems and their bases, the spherical-polar- and the cylindrical-coordinate meshes, tensors of first, second and higher orders,algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, application of tensors in modeling anisotropic systems, study of physical tensors (moment of inertia, index of refraction, etc.), diagnolization of inertia tensor as aligning coördinate frame with natural symmetries of the system.

Recommended Books:

[1] Bourne DE, Kendall PC, *Vector Analysis and Cartesian Tensors* (2^{nd} edition), Thomas Nelson.

[2] Shah NA, *Vector and Tensor Analysis*, 2005, A-One Publishers, Lahore.

[3] Smith GD, *Vector Analysis*, Oxford University Press, Oxford.

[4] Spiegel MR, *Vector Analysis*, 1974, McGraw Hill, New York.

MAT-6314 Computing Tools for Mathematicians 3(2-1)

Prerequisite(s): Programming Languages for Mathematicians

Specific Objectives of the Course:

The purpose of this course is to teach students the use of mathematical software like MATLAB, MAPLE, MATHEMATICA for solving computationally-difficult problems in mathematics. The student shall become well versed in using at least one of the mathematical software and shall learn a number of techniques that are useful in calculus as well as in other areas of mathematics.

Course Outline:

The contents of the course are not fixed, however the following points should be kept in mind while teaching the course. The course should be taught in a computer lab setting. Besides learning to use the software, the students must be able to utilize the software to solve computationally difficult problems in calculus and other areas of mathematics. At the end of the course, the students should have a good command on at least two of the three programs mentioned above.

Recommended Books:

[1] Etter DM, Kuncicky D, Hull D, Introduction to MATLAB 6, 2001, Prentice Hall, Englewood Cliffs, NJ, USA.

[2] Garvan F, *The Maple Book*, 2002, Chapman & Hall/CRC.

[3] Kaufmann S, Mathematica as a Tool: An Introduction with Practical Examples, 1994, Springer, New York.

MAT-6315 **Number Theory** **3(3-0)**

Prerequisite(s): Calculus-I

Specific Objectives of the Course:

This course shall assume no experience or background in number theory or theoretical mathematics. The course introduces various strategies for composing mathematical proofs.

Course Outline:

Divisibility, Euclidean algorithm, GCD and LCM of 2 integers, properties of prime numbers, fundamental theorem of arithmetic (UFT), congruence relation, residue system, Euler's phi-function, solution of system of linear congruences, congruences of higher degree, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem and applications, primitive roots and indices; integers belonging to a given exponent (mod p), primitive roots of prime and composite moduli, indices, solutions of congruences using indices., quadratic residues, composite moduli, quadratic residues of primes, the Legendre symbol, the Quadratic reciprocity law, the Jacobi symbol, Diophantine equations.

Recommended Books:

[1] Burton DM, Elementary Number Theory, Allyn and Bacon Grosswald E, Topics from the Theory of Numbers, The Macmillan Company.

[2] LeVeque WJ, Topics in Number Theory, Addison-Wesley, Reading, Ma, USA.

[3] Niven I, Zuckerman HS, An Introduction to The Theory of Numbers, Wiley Eastern.

[4] Rosen KH, Elementary Number Theory and its Applications (4th edition), 2000, Addison-Wesley, Reading, Ma, USA (suggested text).

Courses for Semester-VI

MAT-6401 **Numerical Analysis-II** **3(3-0)**

Prerequisite(s): Numerical Analysis-I

Specific Objectives of the Course:

The most phenomena in our World are essentially non-linear or desire by non-linear equations may be PDE and ODE since that appearance of high performance digit computers it becomes easier to solve the problem .However, Generally Spacing it is still difficult to obtained or get an analytical approximations then a numerical one of a given non-Linear problem. The numerical techniques generally can be applied to non-linear problems in complicated computational domain. This is obvious of advantage on numerical methods over analytical one that often handle non-linear problem in simple domain. Numerical method gives discontinuous points of a curve. Thus it is often costly or time consuming to get a complete of those results. Besides from numerical results it is hard to have whole and essential understanding of non-linear problems. A number of software package has been developed to produce symbolic mathematical computations such as Mathematica and Matlab.

Course Outline:

Osculating polynomials; Differentiation and integration in multidimensional; Predicator methods; Modified Euler's Method; Truncation error and stability; The Taylor Series method; Runge-Kutta methods; Differential equations of higher order system of differential equations; Runge-Kutta methods; Shooting methods; Finite difference methods; Elliptic, hyperbolic and parabolic equations; Explicit and implicit finite difference methods; Stability; Convergence and consistency analysis; The method of characteristic; Estimation of eigenvalues and corresponding error bounds; Gerschgorin's theorem and its applications; Power method; Shift of origin; Deflation method for the subdominant eigenvalues.

Recommended Books:

[1] Conte SD, De Boor, Elementary Numerical Analysis, 1972, McGraw-Hill.

[2] Gerald CF, Applied Numerical Analysis, 2006, Addison Wesley.

[3] Froberg CE, Introduction to Numerical Analysis, 1972, Addison Wesley.

[4] Gourlay AR, Watson GA, Computational Methods of Matrix Eigen Problems, 1973, John Wiley & Sons.

[5] Smith GD, Numerical Solution of Partial Differential Equations, 1986, Oxford University Press.

[6] Mitchel AR, Griffith DE, The Finite Difference Methods in Partial Differential Equations, 1980, John Wiley & Sons.

MAT-6402 **Differential Geometry-II** **3(3-0)**

Prerequisite(s): Differential Geometry-I

Specific Objectives of the Course:

Differential geometry is a main branch of analysis and geometry. It is impossible to solve any problem of physical life without clear understanding of basic ideas of differential geometry. Particularly, modern physics is written in the language of differential geometry. This course aims to introduce this language.

Course Outline:

Extension of analytical geometry to n-dimensional flat space, Cartesian tensors, Curved space and manifolds, Tangent and cotangent spaces, Vector fields and their flows, Lie derivatives of vector fields and dual vector fields, Metric connection, Tensors on manifolds and their Lie and covariant differentials, Killing vector fields, Curvature tensor and the Bianchi identities, Geodesics and the exponential map, Heuristic to integration on manifolds.

Recommended Books:

[1] Laugwitz D, Differential and Riemannian Geometry, 1970, Academic Press.

[2] Livelock D, Rund H, Tensors: Differential forms and Variational Principles, 1975, John Wiley.

[3] Eisenhart LP, Riemannian Geometry, 1964, Princeton University Press.

[4] Eisenhart LP, An Introduction to Differential Geometry with use of the Tensor Calculus, 1947, Princeton University Press.

MAT-6403 **Mathematical Physics** **3(3-0)**

Prerequisite(s): Partial Differential Equations

Specific Objectives of the Course:

There are several distinct branches of mathematical physics; these roughly correspond to particular historical periods. In this course we shell study the Lap lace transform, Fourier transform and variational techniques. Laplace transform reduces the solution of an ordinary differential equation to an algebraic equation. In fact this method has a particular advantage in finding the general solution and the using for evaluating the arbitrary constant with appropriate initial conditions without finding the general solution and then using initial conditions for evaluating the arbitrary constants. Also when the Laplace transform technique is applied to partial differential equations, it reduces the number of independent variables by one. Fourier transform techniques have been widely used to solve problems involving semi-infinite or totally infinite range of variables or unbounded regions. The one of most interesting method used in mathematical physics is the calculus of variations. The theory of partial differential equations, the related areas of variational calculus and Fourier analysis are closely related with mathematical physics.

Course Outline:

Definitions and properties of Laplace transforms with proofs, The inversion problems, Convolution and inversion theorem with illustrative examples. Applications of Laplace transforms to ordinary and partial differential equations, Definition and basic properties of Fourier Transforms with proofs, Fourier integrals, Convolution theorem. Persaval's theorems, Fourier sine and cosine transforms with illustrative examples, Fourier sine and cosine transforms of derivatives, Applications of Fourier transforms to boundary value problems.

Recommended Books:

[1] Butkov EL, Mathematical physics, Addison-Wesley.

[2] Sagan H, Boundary and Eigen value Problems in Mathematical Physics.

[3] Arfken G, Mathematical Methods for Physics, Academic press.

MAT-6404 **Fluid Mechanics-II** **3(3-0)**

Prerequisite(s): Fluid Mechanics-I

Specific Objectives of the Course:

Fluid mechanics is an exciting and fascinating subject with unlimited practical applications ranging from microscopic biological systems to modern technological developments in engineering and industry. Fluid mechanic has historically been one of the most challenging subjects for Graduate/Undergraduate students. Fluid mechanics is a very broad filed. A small library of books would be required to cover all of the topics that could be included in it. In this course we shall be interested mainly in flows of interest to daily life science/engineering problems but even that is very broad area so we shall classify the types of problems that may be encountered. The original aims of this course to develop the basic ideas/concepts, fundamental laws, equations (Constitutive) for viscous / Newtonian fluids. After presenting the basic concepts of fluid mechanics we than discuss how mathematical models for physical/engineering problems are prepare and how to interpret the result obtain from the analysis of such models than a systematic problem-solving techniques/ method that can be used to solve those problems in detail.

Course Outline:

Constitutive equations, Navier-Strokes equations, Exact solutions of Navier-Strokes equations, Steady unidirectional flow, Poiseuille flow, Couette flow , Unsteady unidirectional flow , Sudden motion of a plane boundary in a fluid at rest , Flow due to an oscillatory boundary , Equations of motion relative to a rotating system, Ekman flow , Dynamical similarity and the Reynold's number, Flow over a flat plate (Blasius solution). Reynold 's equations of turbulent motion.

Recommended Books:

[1] Landau LD, Lifshitz EM, Fluid Mechanics, 1966, Pergamon Press.

[2] Batchelor GK, An introduction to Fluid Dynamics, 1969, Cambridge University Press.

[3] Jaunzemies W, Continuum Mechanics, 1967, Macmillan Company.

[4] Milne Thomson, Theoretical Hydrodynamics, 1967, Macmillan Company.

[5] Schlichting H, Boundary Layer Theory, 1979, McGraw Hill.

[6] Streeter, Hand Book of Fluid Dynamics, McGraw Hill.

[7] Charlton F, Textbook of fluid Dynamics, 1967, D. Van Nostrand Co. Ltd.

MAT-6405 **Quantum Mechanics** **3(3-0)**

Prerequisite(s): Classical Mechanics

Specific Objectives of the Course:

The mathematical formulation of quantum mechanics is abstract and its implications are often non-intuitive. The centerpiece of this mathematical system is the wave function. The wave function is a mathematical function of time and space that can provide information about the position and momentum of a particle, but only as probabilities, as dictated by the constraints imposed by the uncertainty principle. Mathematical manipulations of the wave function usually involve the bracket notation, which requires an understanding of complex numbers and linear functional. Many of the results of Quantum Mechanics can only be expressed mathematically and do not have models that are as easy to visualize as those of classical mechanics. For instance, the ground state in quantum mechanical model is a non-zero energy state that is the lowest permitted energy state of a system, rather than a more traditional system that is thought of as simple being at rest with zero kinetic energy.

Course Outline:

Basic postulates of quantum mechanics, State vectors, Formal Properties of quantum mechanical operators. Eigenvalues and Eigen-states, Simple harmonic oscillator, Schordinger representation, Heisenberg equation of motions, Schrodinger equation, Potential step, Potential barrier, Potential well, Orbital angular momentum motion in a centrally symmetric field, Hydrogen atom, Matrix representation of angular momentum and spin, Time independent perturbation theory, Degeneracy, The stark effect, Introduction to relativistic quantum mechanics.

Recommended Books:

[1] Fayyazuddin, Riazuddin, Quantum Mechanics, 1990, World Scientific.

[2] Merzebacher E, Quantum Mechanics, 1970, John Wiley.

[3] Liboff RL, Introduction Quantum Mechanics, 1991, Addison-Wesley.

[4] Dirac PMA, Principles of Quantum Mechanics, 1985, Oxford University Press.

MAT-6406 Integral Equations 3(3-0)

Prerequisite(s): Ordinary Differential Equations

Specific Objectives of the Course:

Many physical problems which are usually solved by differential equation methods can be solved more effectively by integral equation method. Indeed, the latter have been appearing in current literature with increasing frequency and have provided solutions to problems heretofore not solvable by standard methods of differential equations, and the type of solutions explored in this course will be useful particularly in applied mathematics, theoretical mechanics, and mathematical physics. If the kernel is separable, the problem of solving an integral equation of second kind reduces to that of solving an algebraic system of equations. Unfortunately, integral equations with degenerate kernel do not occur frequently in practice. But they are easily treated, and furthermore, the results derived in this course for such questions lead to better understanding of integral equations of more general type, it is worthwhile to study them. When an integral equation cannot be solved in closed form, then recourse has to be taken to approximate methods can be applied with confidence only if the existence of the solution is assured in advance. The Fredholm theory included in this course provides such an assurance. We shall study the Hilbert-Schmidt theory, the Wiener-Hopf technique which is very useful in solving problems in science and engineering.

Course Outline:

Integral equation formulation of boundary value problems, Classification of integral equations, Method of successive approximation, Hilbert-Schmidt theory, Schmidt's solution of non-homogeneous integral equations, Fredholm theory, Care of multiple roots of characteristic equation, Degenerate kernels, Introduction to Wiener-Hopf technique.

Recommended Books:

[1] Lovitt WV, Linear Integral Equations, 1950, Dover Publication.

[2] Smith F, Integral Equations, 2003, Cambridge University Press.

[3] Tricomi FG, Integral Equations, 1957, Interscience.

[4] Noble B, Methods Based on the Wiener-Hopf Technique, 1958, Pergamon Press.

[5] Abdul JJ, Introduction to Integral Equations with Applications, 1985, Marcel Dekker Inc. New York.

MAT-6407 An Introduction to Convex Analysis 3(3-0)

Prerequisite(s): Functional Analysis-I

Specific Objectives of the Course:

The main purpose of this course is to introduce the convexity. The prerequisites are mainly linear algebra and linear programming (LP) including the duality theorem and the simplex algorithm. The second, and final, part of the course is to go into convexity. The plenty of material in convexity is presented in this course.

Course Outline:

The basic concepts, Convex hulls and Caratheodory's theorem, Projection and separation, Representation of convex sets, Convex functions, Nonlinear and convex optimization.

Recommended Books:

[1] Jean-Baptiste Hiriart-Urruty, *Fundamentals of Convex Analysis*, 2003, Springer.

[2] Magaril-Ilyaev GG, Tikhomirov VM, *Convex Analysis*: Theory and Applications, 2003, AMS.

MAT-6408 Calculus of Variation & Optimal Control 3(3-0)

Prerequisite(s): Optimization Theory

Specific Objectives of the Course:

A huge amount of problems in the calculus of variations have their origin in physics where one has to minimize the energy associated to the problem under consideration. Nowadays many problems come from economics. Here is the main point that the resources are restricted. There is no economy without restricted resources. The calculus of variations is concerned with the construction of optimal shapes, states, or processes where the optimality criterion is given in the form of an integral involving an unknown function. The task of the calculus of variations then to demonstrate the existence and to deduce the properties of some function that realizes the optimal value for this integral. Such problems occur in many-fold applications, in particular physics, engineering, economics and variational integral may represents some action, energy, or cost functional. The calculus of variations also has deep and important connections with other fields of mathematics. For instance, in geometrically defined classes of objects, a variational principle often permits the selection of a unique optimal representative and the properties of can frequently be used to much advantage to deduce additional information about its class. For these reasons, the calculus of variations is a rich mathematical subject.

Course Outline:

Variation of the functionals, Euler-Lagrange equation and its particular cases, Lagrange problem with free end points, Lagrange problem with more than one functionals, Variational problems with constraints,from Calculus of Variations to Optimal Control, The Maximum Principle, The Hamilton-Jacobi-Bellman equation, The Linear Quadratic Regulator.

Recommended Books:

[1] Moser J, *Selected Chapters in the Calculus of Variations*, 2003, Birkhauser-Verlag, Switzerland.

[2] Liberzon D, *Calculus of Variations & Optimal Control Theory*, 2012, Princeton University Press.

MAT-6409 Dynamical Systems 3(3-0)

Prerequisite(s): Ordinary Differential Equations, Linear Algebra

Specific Objectives of the Course:

After taking this course it is expected that the students will learn about the linear and nonlinear dynamical systems. They will be able to construct and analyze the models of real time-dependent systems in several different areas of study. Moreover, this course will be helpful to use Mathematica for the investigation of different properties of dynamical systems.

Course Outline:

Introduction: Preliminary ideas, Autonomous equations, Autonomous systems in plane, Linear systems: Linear changes of variables, Similarity types for 2×2 real matrices, Phase portraits for canonical systems in the plane, Classification of simple linear phase portraits in the plane. Nonlinear systems in the plane: Local and global behavior, Linearization at a fixed point, The linearization theorem, Non-simple fixed points, Stability of fixed points, Ordinary points and global behavior. Applications: Linear models, Nonlinear models, Relaxation oscillation, Piecewise modeling. Dynamical systems with Mathematica: Differential equations, Planar systems, Interacting species.

Recommended Books:

[1] Lynch S, Dynamical systems with Applications using Mathematica, 2007, Birkhauser Boston.

[2] Alligood TK, Sauer DT, Yorke AJ, Chaos: An Introduction to Dynamical Systems, 1996, Springer.

[3] Arrowsmith KD, Place MC, Dynamical Systems, Differential Equations, Maps and Chaotic Behavior, 1992, Chapman & Hall.

MAT-6410 **Computer Language C/C++** **3(3-0)**

Prerequisite(s): Programming Languages for Mathematicians

Specific Objectives of the Course:

The main objectives of this course are:

To present the material one simple step at a time, so the students can easily digest each concept before moving on. To explore the issues of when and how to use in lines, references, operator overloading, inheritance and dynamic objects. To introduce advanced topics such as the proper use of templates, exceptions and multiple inheritance.

Course Outline:

Introduction to objects, Making & using objects, The C in C++, Data abstraction, Hiding the implementation, Initialization & cleanup, Function overloading & default arguments, Constants, Inline functions, Name control, References & the copy-constructor, Operator overloading, Dynamic object creation, Inheritance & composition, Polymorphism & virtual functions, Introduction to templates.

Recommended Books:

[1] Ackel B, Thinking in C++, 2000, Prentice Hall.

[2] Anderson, C++ Programming & Fundamental Concepts, Prentice Hall.

[3] Lam, A Jump Start Course in C++ Programming, Wiley.

MAT-6411 **Electromagnetism** **3(3-0)**

Prerequisite(s): Fluid Mechanics-I

Specific Objectives of the Course:

Electromagnetism is the branch of science concerned with the forces that occur between electrically charged particles. In electromagnetic theory these forces are explained using electromagnetic fields. Electromagnetic force is one of the four fundamental interactions in nature, the other three being the strong, the weak interaction and gravitation. Electromagnetism is the interaction responsible for practically all the phenomena encountered in daily life, with the exception of gravity. Ordinary matters takes its form as a result of intermolecular forces between individual molecules in matter. Electromagnetism manifests as both electric fields magnetic fields. Both fields are simply different aspects of electromagnetism and hence intrinsically related. Thus, a charging electric field generates a magnetic field; conversely a charging magnetic field generates an electric field. This effect is called electromagnetic induction, and is the basis of operations for electrical generators, induction motors and transformers.

Course Outline:

Electrostatics and the solution of problems in vacuum and in media, Electrostatic energy, Electric currents, The magnetic fields of steady currents, Magnetic properties of matter and its Applications, Magnetic energy, Electromagnetic induction, Maxwell's Equations and Applications, Boundary value potential problems in two dimensions and Applications, Electromagnetic waves, Radiation, Motion of electric charges and their Applications.

Recommended Books:

[1] Reitz JR, Milford FJ, Foundation of Electromagnetic Theory, 1969, Addison-Wesley.

[2] Panofsky KH, Philips M, Classical Electricity and Magnetism, 1962, Addison-Wesley.

[3] Corson D, Lerrain P, Introduction to Electromagnetic Fields and Waves, 1962, Freeman.

[4] Ferraro VCA, Electromagnetic Theory, 1968, The Athlone Press.

MAT-6412 Special Theory of Relativity 3(3-0)

Prerequisite(s): Classical Mechanics

Specific Objectives of the Course:

General relativity or the general theory of relativity is the geometric theory of gravitation published by Albert Einstein in 1915. It is the current description of gravitation in modern physics. It unifies special relativity and Newton's law of universal gravitation, and describes gravity as a geometric property of space and time. In particular, the curvature of space and time is directly related to the four-momentum (mass-energy and linear momentum) of whatever matter and radiation are present. The relation is specified by the Einstein field equations, a system of partial differential equations. To learn general relativity which differ significantly from those of classical physics, especially concerning the passage of time, the geometry of space, the motion of bodies in free fall, and the propagation of light.

Course Outline:

Historical background and fundamental concepts of special theory of Relativity, Lorentz Transformations (For motion along one axis), Length contraction, Time dilation and Simultaneity, Velocity addition formulae 3-dimensional Lorentz transformation, Introduction to 4-vector formalism, Lorentz transformations in the 4-vector formalism, The Lorentz and Poincare groups, Introduction to classical Mechanics, Minkowski space-time and null con, 4-velocity, 4-momentum and 4-force, Application of special relativity to Doppler shift and Compton effect, Particle scattering, Binding energy, Particle production and decay, Electromagnetism in relativity. Electric current, Maxwell's equations and electromagnetic waves, 4-vector formulation of Maxwell's equations, Special relativity with small acceleration.

Recommended Books:

[1] Qadir A, Relativity: An Introduction to the Special Theory, 1989, World Scientific.

[2] Goldstein H, Classical Mechanics, 1962, Addison-Wesley, New York.

[3] Jackson JD, Classical Electrodynamics, 1962, John Wiley, New York.

[4] Rindler W, Essential Relativity, 1977, Springer-Verlag.

MAT-6413 **Topology-II** **3(3-0)**

Prerequisite(s): Topology-I

Specific Objectives of the Course:

This is a continuation of the study of topology and how it extends the ideas of geometry, place, location and analysis, as well as some of the newer applications of topology.

Course Outline:

Compactness in metric spaces, Limit point compactness, Sequential compactness and their various characterization, Equivalence of different notions of compactness. Connectedness with examples, Various characterizations of connectedness and its Application, Connectedness and T_2-spaces, Local connectedness, Path-connectedness, Components and its Application. Homotopic maps with examples, Homotopic paths, Loop spaces and its Application, Fundamental groups, Covering spaces, The chain complexes, Notion of homology.

Recommended Books:

[1] Greenberg MJ, Algebraic Topology, A First Course, 1967, The Benjamin/Commings publishing Company.

[2] Wallace AH, Algebraic Topology, Homology and Cohomology, 1968, New York.

[3] Gemignani MC, Elementary Topology, 1972, Addison-Wesley Publishing Company.

[4] Ahmad B, Introduction to General Topology, 2004, Ideal Publishers.

MAT-6414 **Theory of Elasticity** **3(3-0)**

Prerequisite(s): Classical Mechanics

Specific Objectives of the Course:

Linear elasticity is the mathematical study of how solid objects deforms and become intemally stressed due to prescribed condition. It relies upon the continuum hypothesis and in applicable at macroscopic (and sometime microscopic) length seals. It is a branch of continuum mechanics. The fundamental (linearizing) assumptions of linear elasticity are infinitesimal strain or small deformations and linear relationship between stress & rate of strain. In addition linear elasticity is only valid for stress state, that do not produce yielding. These assumptions are reasonable for many engineering materials. Linear elasticity in therefore used extensively in structural analysis and engineering design. We hope that at the end of the course the student will understand the concepts / basic of elasticity and have a working knowledge as well as creative thinking.

Course Outline:

Cartesian tensors, Analysis of stress and strain, Generalized Hooke’s law, Crystalline structure, Point groups of crystals, Reduction in the number of elastic moduli due to crystal symmetry, Equations of equilibrium, Boundary conditions, Compatibility equation, plane stress and plane strain problems, Two dimensional problem in rectangular and polar co-ordinates, Torsion of rods and beams.

Recommended Books:

[1] Sokolinikoff, Mathematical Theory of Elasticity, McGraw-Hill, New York.

[2] Dieulesaint E, Royer D, Elastic Waves in Solids, 1980, John Wiley and Sons, New York.

[3] Funk YC, Foundations of Solid Mechanics, 1965, Prentice-Hall, Englewood Cliffs.

MAT-6415 **Decomposition of Modules** **3(3-0)**

Prerequisite(s): Ring Theory

Specific Objectives of the Course:

In graph theory, the modular decomposition is a decomposition of an undirected graph into subsets of vertices called modules. A module is a generalization of connected component of a graph. Unlike connected components, however, one module can be proper subset of another. Modules therefore lead to a recursive (hierarchical) decomposition of graph, instead of just a partition. For each undirected graph, this decomposition is unique. At the end of the course we expect that the students understand the concepts of Decomposition of Modules.

Course Outline:

Rings and modules with examples, decomposition of modules and their Applications, Decomposition theorem, The primary Decomposition theorem, The primary Decomposition, Abelian groups as z-modules, Abelian groups, Sylow's theorem, Linear transformation and matrices, Invariants and the Jordan canonical form, The rational canonical form theorem (linear transformation version), The Jordan canonical form theorem, Conjugacy classes in general linear groups.

Recommended Books:

[1] Blyth T, Modules theory, 1977, O.U.P., Oxford.

[2] Hartley B, Hawkes T, Rings, Modules and linear Algebra, Chapman G. Lecture Notes on Modules, Michigan University Press.

MAT-6416 **Mathematical Statistics-II** **3(3-0)**

Prerequisite(s): Mathematical Statistics-I

Specific Objectives of the Course:

In the course "Probability Theory" the students learnt how to set up mathematical models of processes and systems that are affected by *chance*. In the present course the students would learn how to check these models against reality, to determine whether they are reliable/accurate enough for practical purposes or otherwise. This helps in making predictions and decisions.

Course Outline:

Sampling theory: sampling distributions; sampling procedures; estimation of parameters: estimation of mean, variance; confidence intervals; decision theory: hypothesis testing and decision making; types of errors in tests; quality control; control charts for mean, standard deviation, variance, range; goodness of fit, chi-square test; regression analysis; method of least squares; correlation analysis.

Recommended Books:

[1] DeGroot MH, Schervish MJ, Probability and Statistics (3rd edition), 2002, Addison-Wesley, Reading, Ma, USA (suggested text).

[2] Johnson RA, Probability and Statistics for Engineers, 1994, Prentice-Hall, Englewood Cliffs, NJ, USA.

[3] Papoulis A, Probability, Random Variables, and Stochastic Processes, (3rd edition), 1991, McGraw Hill, New York.