THE UNIVERSITY OF POONCH RAWALAKOT

AZAD JAMMU AND KASHMIR

Scheme of Study for M. Phil Program in Mathematics

M.Phil. Program

Duration of program:	4-6 semesters
Courses:	24 credits
Thesis (MATH-796):	06 credits
Conference/Seminar/Reading (MATH-795):	01 credit
Total credits:	31 credits
Comprehensive Oral Examination:	Satisfactory/Unsatisfactory basis

Semester-I

Note: Any four courses from the following elective courses.

Course Code	Course Title	Credit Hours
MATH-701	Advanced Functional Analysis-I	3+0
MATH-703	Basics of the Theory of Fluids	3+0
MATH-705	Advanced Partial Differential Equations	3+0
MATH-707	Theory of Differential Equations	3+0
MATH-709	Fixed Point Theory	3+0
MATH-711	Advanced Numerical Analysis	3+0
MATH-713	Computing Techniques for Dynamical Systems	3+0

Semester-II

Note: Any four courses from the following elective courses.

Course Code	Course Title	Credit Hours
MATH-702	Advanced Functional Analysis-II	3+0
MATH-704	Advanced Fluid Dynamics	3+0
MATH-706	Numerical Solutions of Partial Differential Equations	3+0
MATH-708	Advances in Discrete Mathematics and Applications	3+0
MATH-710	Fuzzy Analysis	3+0
MATH-712	Advanced Integral Equations	3+0
MATH-714	Fractional Differential Equations	3+0

Course Contents for M. Phil Program

The contents for the courses of Mathematics for the said program are presented for discussion.

MATH–701 ADVANCED FUNCTIONAL ANALYSIS-I Credits: 3+0

Vector Spaces, Normed Spaces, Banach Spaces, Properties of Normed Spaces, Finite Dimensional Normed Spaces and Subspaces, Compactness and Finite Dimension, Linear Operators, Bounded and Continuous Linear Operators, Linear Functionals, Linear Operators and Functionals on Finite Dimensional Spaces, Normed Spaces of Operators, Dual Spaces, Inner Product Spaces, Hilbert Spaces, Further Properties of Inner Product Spaces, Orthogonal Complements and Direct Sums, Orthonormal Sets and Sequences, Series Related to Orthonormal Sequences and Sets, Total Orthonormal Sets and Sequences, Legendre and Hermite Polynomials, Representation of Functionals on Hilbert Spaces, Hilbert-Adjoint Operator, Self-Adjoint, Unitary and Normal Operators, Zorn's Lemma, Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector Spaces and Normed Spaces, Application to Bounded Linear Functionals on the Space C[a, b], Adjoint-Operator, Reflexive Spaces, Category Theorem, Uniform Boundedness Theorem, Strong and Weak Convergence, Convergence of Sequences of Operators and Functionals, Application to Summability of Sequences, Numerical Integration and Weak-Convergence, Open Mapping Theorem, Closed Linear Operators, Closed Graph Theorem.

RECOMMENDED BOOKS:

1. Kreyszig E., Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1989.
2. Cohen G.L., A Course in Modern Analysis and Its Applications, Cambridge University Press, 2003.
3. Bollobás B., Linear Analysis: An Introductory Course, Cambridge University Press, 2nd edition, 2012.
4. Giles G.R., Introduction to the Analysis of Normed Linear Spaces, Cambridge University Press, 2000.
5. Cheney W., Analysis for Applied Mathematics**,** Springer-Verlag, New York, 2001.
6. Hirsch F., Lacombe G., Levy S., Elements of Functional Analysis**,** Springer–Verlag, 2009.

MATH-703 BASICS OF THE THEORY OF FLUIDS Credits: 3+0

Newtonian Fluids: Some examples of viscous flow phenomena, Properties of fluids, Boundary conditions, Equation of continuity, Navier Stoke equations, Energy equation, Boundary conditions, Orthogonal coordinate systems, Dimensionless parameters, Velocity considerations, Two-dimensional considerations, Stream functions, Couette flows, Poiseuille flow, Unsteady duct flows, Similarity solutions, Some exact analytic solutions, Laminar boundary layers equations, Similarity solutions, Two-dimensional solutions, Thermal boundary layer.

The concept of small disturbance stability, Linearized stability, Parametric effects in the linear stability theory, Transition to turbulences, Boundary layer equation in plane flow, General solution and exact solutions of the boundary layer equations, Thermal boundary layers without coupling of velocity field to the temperature field, Boundary layer equations for the temperature field, Forced convection, Effect of Pr number, Similar solution of the thermal boundary layers, Thermal boundary layer with coupling of velocity field to the temperature field, Boundary layer with moderate wall heat transfer, Natural convection effect of dissipation, Indirect natural convection, Mixed convection, Different kinds of boundary layer control, Continuous suction and blowing, Massive suction and blowing, Similar solutions.

RECOMMENDED BOOKS:

1. White F.M., Viscous Fluid Flow, McGraw Hill Inc., 1991.
2. Schlichting H., Gersten K., Boundary-Layer Theory, Springer, 1991.
3. Davidson P.A., An Introduction to Magnetohydrodynamics, Cambridge University Press, 2001.
4. Truesdell C., Rajagopal K.R., An Introduction to the Mechanics of Fluids, Birkhauser Boston, 2009.
5. Morrison F.A., An Introduction to Fluid Mechanics, Cambridge University Press, 2013.
6. Durst F., Fluid Mechanics: An Introduction to the Theory of Fluid Flows, Springer, 2008.
7. Batchelor G.K., An Introduction to Fluid Dynamics, Cambridge University Press, 2000.
8. Biringen S., Chow C.-Y., An Introduction to Computational Fluid Mechanics by Example, John Wiley & Sons, Inc., 2011.

MATH-705 ADVANCED PARTIAL DIFFERENTIAL EQUATIONS Credits: 3+0

First-order equations: Introduction, Quasilinear equations, The method of characteristics, Examples of the characteristics method, The existence and uniqueness theorem, The Lagrange method, Conservation laws and shock waves, The Eikonal equation, General nonlinear equations. **Second-order linear equations in two independent variables:** Canonical form of hyperbolic equations, Canonical form of parabolic equations, Canonical form of elliptic equations. **The one-dimensional wave equation:** Canonical form and general solution, The Cauchy problem and d'Alembert's formula, Domain of dependence and region of influence, The Cauchy problem for the nonhomogeneous wave equation.

The method of separation of variables: Heat equation, homogeneous boundary condition, Separation of variables for the wave equation, Separation of variables for nonhomogeneous equations, The energy method and uniqueness, Further applications of the heat equation. **Sturm–Liouville problems and eigenfunction expansions:** The Sturm–Liouville problem, Inner product spaces and orthonormal systems, The basic properties of Sturm–Liouville eigenfunctions and eigenvalues, Nonhomogeneous equations, Nonhomogeneous boundary conditions. **Elliptic equations:** Basic properties of elliptic problems, The maximum principle, Applications of the maximum principle, Green's identities, The maximum principle for the heat equation, Separation of variables for elliptic problems, Poisson's formula. **Green's functions and integral representations:** Green's function for Dirichlet problem in the plane, Neumann's function in the plane, The heat kernel. **Equations in high dimensions:** First-order equations, Classification of second-order equations, The wave equation in $\mathbb{R}^2$ and $\mathbb{R}^3$, The eigenvalue problem for the Laplace equation, Separation of variables for the heat equation, Separation of variables for the wave equation, Separation of variables for the Laplace equation, Schrodinger equation for the hydrogen atom, Musical instruments, Green's functions in higher dimensions, Heat kernel in higher dimensions.

RECOMMENDED BOOKS:

1. Pinchover Y., Rubinstein J., An Introduction to Partial Differential Equations, Cambridge University Press, 2005.
2. Zachmanoglou E.C., Thoe D.W., Introduction to Partial Differential Equations with Applications, Dover Publications, Inc., New York, 1986.
3. Adzievski K., Siddiqi A.H., Introduction to Partial Differential Equations for Scientists and Engineers using Mathematica, CRC Press, 2014.
4. Coleman P.M., An Introduction to Partial Differential Equations with MATLAB, Chapman & Hall/CRC, Boca Raton, London, New York, 2005.
5. Cain J.W., Reynolds A.M., Ordinary and Partial Differential Equations: An Introduction to Dynamical Systems, NC-ND, 2010.

MATH-707 THEORY OF DIFFERENTIAL EQUATIONS Credits: 3+0

First-Order Differential Equations: Basic Results, First-Order Linear Equations, Autonomous Equations, Generalized Logistic Equation, Bifurcation.

Linear Systems: Introduction, The Vector Equations, The Matrix Exponential Function, Induced Matrix Norm, Floquet Theory.

Autonomous Systems: Introduction, Phase Plane Diagrams, Phase Plane Diagrams for Linear Systems, Stability of Nonlinear Systems, Lyapunov Stability Theorem, LaSalle Invariance Theorem, Linearization of Nonlinear Systems, Applications: Prey-predator model, Glycolysis Model, Infectious Diseases Modelling, Competitive Species, Competition and Harvesting, Circuit Theory, The van der Pol Equation, Models in Neurodynamics, Applications in Mechanics, Elementary Properties of the Lorenz System, Dynamics of Rössler system.

Existence and Nonexistence of Periodic Solutions, Three-Dimensional Systems, Differential Equations and Mathematica.

Existence and Uniqueness Theorems: Basic Results, Lipschitz Condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem, Cauchy-Peano Theorem, Extendability of Solutions, Basic Convergence Theorem.

RECOMMENDED BOOKS:

1. Walter G.K., Allan C.P., The Theory of Differential Equations: Classical & Qualitative, Springer, New York, 2010.
2. Layek G.C., An Introduction to Dynamical Systems and Chaos, Springer, 2015.
3. Brauer F., Nohel J.A., The Qualitative Theory of Ordinary Differential Equations: An Introduction, Dover, 1989.
4. Perko L., Differential Equations and Dynamical Systems, 3rd edition, Springer, New York, 2001.
5. Hirsch M.W., Smale S., Devaney R.L., Differential Equations, Dynamical Systems, and an Introduction to Chaos, Elsevier, 2004.
6. Agarwal R.P., O’Regan D., An Introduction to Ordinary Differential Equations, Springer, 2008.

MATH-709 **FIXED POINT THEORY** **Credits: 3+0**

The Banach fixed-point theorem and iterative methods: The Banach fixed-point theorem, Continuous dependence on a parameter, The significance of Banach fixed point theorem, Applications to nonlinear equations, Accelerated convergence and Newton's method, The Picard-Lindelof theorem, The main theorems for iterative methods for linear operator equations, Applications to systems of linear equations, Applications to linear integral equations. **The Schauder fixed-point theorem and compactness:** Extension theorem, Retracts, The Brouwer fixed-point theorem, Existence principle for system of equations, Compact operators, Schauder fixed-point theorem, Peano's theorem, Integral equations with small parameters, Systems of integral equations and semilinear differential equations, Existence principle for system of inequalities. **Applications of the fundamental fixed-point principles:** Ordinary differential equations in Banach spaces, Integration of vector functions of one real variable, Differentiation of vector functions of one real variable, Generalized Picard-Lindelof theorem, Generalized Peano theorem, Gronwall's lemma, Stability of solutions and existence of periodic solutions, Stability theory and plane vector fields, Caristi's fixed-point theorem. **Set-valued maps:** Basic concepts and definitions, Continuity of set-valued maps, Nadler's fixed-point theorem, Some fixed-point theorems for set-valued maps. **Ekeland's variational principle and its applications:** Application to fixed-point theorems, Applications to optimization, Equilibrium problems and extended Ekeland's variational principle.

RECOMMENDED BOOKS:

1. Zeidler E., Nonlinear Functional Analysis and its Applications: I: Fixed-Point Theorems, Springer-Verlag, New York, 1986.
2. Qamrul H.A., Metric Spaces: Including Fixed Point Theory and Set-valued Maps, Alpha Science International Ltd., UK, 2010.
3. Agarwal R.P., O'Regan D., Sahu D.R., Fixed Point Theory for Lipschitzian-type Mappings with Applications, Springer, Dordrecht, 2009.
4. Kirk W.A., Sims B., Handbook of Metric Fixed Point Theory, Springer, Dordrecht, 2001.
5. Khamsi M.A., Kozlowski W.M., Fixed Point Theory in Modular Function Spaces, Springer International Publishing, Switzerland, 2015.
6. Kirk W., Shahzad N., Fixed Point Theory in Distance Spaces, Springer, Switzerland, 2014.

MATH-711 ADVANCED NUMERICAL ANALYSIS Credits: 3+0

Solution of equations by iteration: Introduction, Simple iteration, Iterative solution of equations, Relaxation and Newton's method, Secant method, Bisection method, Global behavior. **Solution of systems of linear equations:** Introduction, Gaussian elimination, LU factorization, Pivoting, Solution of systems of equations, Computational work, Norms and condition numbers, Hilbert matrix, Least squares method. **Special matrices:** Introduction, Symmetric positive definite matrices, Tridiagonal and band matrices, Monotone matrices. **Simultaneous nonlinear equations:** Introduction, Simultaneous iteration, Relaxation and Newton's method, Global convergence. **Eigenvalues and eigenvectors of a symmetric matrix:** Introduction: The characteristic polynomial, Jacobi's method, The Gerschgorin theorems, Householder's method, Eigenvalues of a tridiagonal matrix, The QR algorithm, The QR factorization revisited, The definition of the QR algorithm, Inverse iteration for the eigenvectors, The Rayleigh quotient, Perturbation analysis. **Polynomial interpolation:** Introduction, Lagrange interpolation, Convergence, Hermite interpolation, Differentiation. **Numerical integration-I:** Introduction, Newton–Cotes formulae, Error estimates, The Runge phenomenon, Composite formulae, The Euler–Maclaurin expansion, Extrapolation methods. **Polynomial approximation in the ∞-norm:** Introduction, Normed linear spaces, Best approximation in the ∞-norm, Chebyshev polynomials, Interpolation. **Approximation in the 2-norm:** Inner product spaces, Best approximation in the 2-norm, Orthogonal polynomials, Comparisons. **Numerical integration-II:** Construction of Gauss quadrature rules, Direct construction, Error estimation for Gauss quadrature, Composite Gauss formulae, Radau and Lobatto quadrature. **Piecewise polynomial approximation:** Linear interpolating splines, Basis functions for the linear spline, Cubic splines, Hermite cubic splines, Basis functions for cubic splines.

RECOMMENDED BOOKS:

1. Suli E., Mayers D.F., An Introduction to Numerical Analysis, Cambridge University Press, 2003.
2. Neumaier A., Introduction to Numerical Analysis, Cambridge University Press, 2001.
3. Hildebrand F.B., Introduction to Numerical Analysis, Dover Publications, Inc., New York, 1987.
4. Gautschi W., Numerical Analysis: an Introduction, Birkhauser, Boston, 1997.
5. Faul A.C., A Concise Introduction to Numerical Analysis, Chapman and Hall/CRC, 2016.
6. Ralston A., Rabinowitz P., A First Course in Numerical Analysis, Dover Publications, 2001.
7. Stoer J., Bulirsch R., Introduction to Numerical Analysis, Springer, 2002.

MATH-713 COMPUTING TECHNIQUES FOR DYNAMICAL SYSTEMS

Credits: 3+0

Differential Equations: Simple Differential Equations and Applications, Applications of Differential Equations, Existence and Uniqueness Theorem, Maple/Mathematica/MATLAB Commands. Planar Systems: Canonical Forms, Eigenvectors Defining Stable and Unstable Manifolds, Phase Portraits of Linear Systems in the Plane, Linearization and Hartman's Theorem, Constructing Phase Plane Diagrams, Maple/Mathematica/MATLAB Commands. Interacting Species: Competing Species, Predator–Prey Models, Other Characteristics Affecting Interacting Species, Maple/Mathematica/MATLAB Commands. Limit Cycles, Hamiltonian Systems, Lyapunov Functions, and Stability, Bifurcation Theory, Three-Dimensional Autonomous Systems and Chaos, Poincare Maps and Non-autonomous Systems in the Plane, Local and Global Bifurcations, Linear Discrete Dynamical Systems, Nonlinear Discrete Dynamical Systems, Chaos Control and Synchronization, Maple/Mathematica/MATLAB Commands.

RECOMMENDED BOOKS:

1. Lynch S., Dynamical Systems with Applications using Maple, Birkhauser Boston, 2010.
2. Lynch S., Dynamical Systems with Applications using MATLAB, Springer International Publishing Switzerland, 2014.
3. Lynch S., Dynamical Systems with Applications using Mathematica, Birkhauser Boston, 2007.
4. Abell M.L., Braselton J.P., Differential Equations with Mathematica, 3rd ed., Academic Press, New York, 2004.
5. Hunt B.R., Lipsman R.L., Osborn J.E., Rosenberg J.M., Differential Equations with Maple, 3rd ed.,Wiley, New York, 2008.
6. Kulenovic M.R.S., Merino O., Discrete Dynamical Systems and Difference Equations with Mathematica, Chapman & Hall, London, 2002.

MATH-702 ADVANCED FUNCTIONAL ANALYSIS-II Credits: 3+0

Banach Fixed Point Theorem, Application of Banach's Theorem to Linear Equations, Applications of Banach's Theorem to Differential Equations, Application of Banach's Theorem to Integral Equations, Approximation in Normed Spaces, Uniqueness, Strict Convexity, Uniform Approximation, Chebyshev Polynomials, Approximation in Hilbert Space, Splines, Spectral Theory in Finite Dimensional Normed Spaces, Basic Concepts, Spectral Properties of Bounded Linear Operators, Further Properties of Resolvent and Spectrum, Use of Complex Analysis in Spectral Theory, Banach Algebras, Further Properties of Banach Algebras, Compact Linear Operators on Normed Spaces, Further Properties of Compact Linear Operators, Spectral Properties of Compact Linear Operators on Normed Spaces, Further Spectral Properties of Compact Linear Operators, Operator Equations Involving Compact Linear Operators, Further Theorems of Fredholm Type, Fredholm Alternative, Spectral Properties of Bounded Self-Adjoint Linear Operators, Further Spectral Properties of Bounded Self-Adjoint Linear Operators, Positive Operators, Square Roots of a Positive Operator, Projection Operators, Further Properties of Projections, Spectral Family, Spectral Family of a Bounded Self-Adjoint Linear Operator, Spectral Representation of Bounded Self-Adjoint Linear Operators, Extension of the Spectral Theorem to Continuous Functions, Properties of the Spectral Family of a Bounded Self-Adioint Linear Operator.

RECOMMENDED BOOKS:

1. Kreyszig E., Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1989.
2. Cohen G.L., A Course in Modern Analysis and Its Applications, Cambridge University Press, 2003.
3. Bollobás B., Linear Analysis: An Introductory Course, Cambridge University Press, 2nd edition, 2012.
4. Giles G.R., Introduction to the Analysis of Normed Linear Spaces, Cambridge University Press, 2000.
5. Cheney W., Analysis for Applied Mathematics, Springer-Verlag, New York, 2001.
6. Hirsch F., Lacombe G., Levy S., Elements of Functional Analysis, Springer–Verlag, 2009.

MATH-704 **ADVANCED FLUID DYNAMICS** **Credits: 3+0**

The basic equations of fluid mechanics: Generalities, Vectors, Tensors, Differentiation of scalars, vectors, and tensors, Frames of reference and coordinate systems, The equation of continuity, The dynamical equation, Pressure, Special forms of the dynamical equation, Energy equations. Purely viscous non-Newtonian constitutive equations: Viscosity of real fluids, Requirements of objectivity for constitutive equations, Reiner-Rivlin fluids, Generalized Newtonian fluids, Laminar flow of generalized Newtonian fluid, The concept of memory for fluid-like materials, Further useful relations of tensor algebra. Kinematics: Deformation and strain, Histories. Time derivatives. Rates of strain, Transformation of tensors and their time derivatives under a change of frame, The convected coordinates approach, Constant stretch history flows. Simple fluid theory: The simple fluid concept, Functional and the principle of fading memory, Simple fluids with fading memory, Mechanical theory, Thermodynamics. Rheometrical flow systems: Introduction, Viscometric flows, Extensional flows, Periodic flows. Constitutive equations: Classification, Differential equations, Integral equations, Rate equations, Illustrations. Fluid mechanics: Flow classification, Dimensional analysis, Secondary and superimposed flows, Flows around submerged objects and boundary layers, Turbulent flows, Extensional flows, Waves and instability.

RECOMMENDED BOOKS:

1. Harris J., Rheology and Non-Newtonian Flow, Longman Inc, New York, 1977.
2. Schowalter W.R., Mechanics of Non-Newtonian Fluids, Pergamon Press, New York, 1978.
3. Bird R.B., Armstrong R.C., Hassager O., Dynamic of Polymeric Liquids, John Wiley & Sons, New York, 1987.
4. Astarita G., Marrucci G., Principles of Non-Newtonian Fluid Mechanics, McGraw-Hill, 1974.
5. Robert W.F., McDonald A.T., Introduction to Fluid Mechanics, John Wiley & Sons, 1995.
6. Truesdell C., Rajagopal K.R., An Introduction to the Mechanics of Fluids, Birkhauser Boston, 2009.
7. Morrison F.A., An Introduction to Fluid Mechanics, Cambridge University Press, 2013.
8. Durst F., Fluid Mechanics: An Introduction to the Theory of Fluid Flows, Springer, 2008.
9. Batchelor G.K., An Introduction to Fluid Dynamics, Cambridge University Press, 2000.

MATH-706 NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Credits: 3+0

Parabolic equations in one space variable: Introduction, A model problem, Series approximation, An explicit scheme for the model problem, Difference notation and truncation error, Convergence of the explicit scheme, Fourier analysis of the error, An implicit method, The Thomas algorithm, The weighted average or θ-method, A maximum principle and convergence for $\mu(1-\theta)\leq 1$, A three-time-level scheme, More general boundary conditions, Heat conservation properties, More general linear problems, Polar co-ordinates, Nonlinear problems. 2-D and 3-D parabolic equations: The explicit method in a rectilinear box, An ADI method in two dimensions, ADI and LOD methods in three dimensions, Curved boundaries, Application to general parabolic problems. Hyperbolic equations in one space dimension: Characteristics, The CFL condition, Error analysis of the upwind scheme, Fourier analysis of the upwind scheme, The Lax–Wendroff scheme, The Lax–Wendroff method for conservation laws, Finite volume schemes, The box scheme, The leap-frog scheme, Hamiltonian systems and symplectic integration schemes, Comparison of phase and amplitude errors, Boundary conditions and conservation properties, Extensions to more space dimensions. Consistency, convergence and stability: Definition of the problems considered, The finite difference mesh and norms, Finite difference approximations, Consistency, order of accuracy and convergence, Stability and the Lax Equivalence Theorem, Calculating stability conditions, Practical (strict or strong) stability, Modified equation analysis, Conservation laws and the energy method of analysis. Linear second order elliptic equations in two dimensions: A model problem, Error analysis of the model problem, The general diffusion equation, Boundary conditions on a curved boundary, Error analysis using a maximum principle, Asymptotic error estimates, Variational formulation and the finite element method, Convection–diffusion problems. Iterative solution of linear algebraic equations: Basic iterative schemes in explicit form, Matrix form of iteration methods and their convergence, Fourier analysis of convergence, Applications, Extensions and related iterative methods, Multigrid method, The conjugate gradient method.

RECOMMENDED BOOKS:

1. Morton K.W., Mayers D.F., Numerical Solution of Partial Differential Equations: An Introduction**,** Cambridge University Press, Cambridge, 2005.
2. Smith G.D., Numerical Solution of Partial Differential Equations: Finite Difference Methods**,** Oxford University Press, 1985.
3. Brenner S., Scott L.R., The Mathematical Theory of Finite Element Methods, Springer, New York, 2002.
4. Briggs W.L., Henson V.E., McCormick S.F., A Multigrid Tutorial, Second edition, SIAM, 2000.
5. Coleman P.M., An Introduction to Partial Differential Equations with MATLAB, Chapman & Hall/CRC, Boca Raton, London, New York, 2005.
6. Adzievski K., Siddiqi A.H., Introduction to Partial Differential Equations for Scientists and Engineers using Mathematica, CRC Press, 2014.

MATH-708 ADVANCES IN DISCRETE MATHEMATICS AND APPLICATIONS

Credits: 3+0

Dynamics of First-Order Difference Equations: Introduction, Linear First-Order Difference Equations, Important Special Cases, Equilibrium Points, The Stair Step (Cobweb) Diagrams, The Cobweb Theorem of Economics, Numerical Solutions of Differential Equations, Euler's Method, A Nonstandard Scheme, Criterion for the Asymptotic Stability of Equilibrium Points, Periodic Points and Cycles, The Logistic Equation and Bifurcation, Equilibrium Points. Linear Difference Equations of Higher Order: Difference Calculus, The Power Shift, Factorial Polynomials, The Anti-difference Operator, General Theory of Linear Difference Equations, Linear Homogeneous Equations with Constant Coefficients, Nonhomogeneous Equations: Methods of Undetermined Coefficients, The Method of Variation of Constants (Parameters), Limiting Behavior of Solutions, Nonlinear Equations Transformable to Linear Equations, Applications, Propagation of Annual Plants, Gambler's Ruin, National Income, The Transmission of Information. Systems of Linear Difference Equations: Autonomous (Time-Invariant) Systems, The Discrete Analogue of the Putzer Algorithm, The Development of the Algorithm for A^n, The Basic Theory, The Jordan Form: Autonomous (Time-Invariant) Systems Revisited, Diagonalizable Matrices, The Jordan Form, Block-Diagonal Matrices, Linear Periodic Systems, Applications. Stability Theory: A Norm of a Matrix, Notions of Stability, Stability of Linear Systems, Nonautonomous Linear Systems, Autonomous Linear Systems, Phase Space Analysis, Liapunov's Direct or Second Method, Stability by Linear Approximation, Applications, One Species with Two Age Classes, Host–Parasitoid Systems, A Business Cycle Model, The Nicholson–Bailey Model, The Flour Beetle Case Study. Higher-Order Scalar Difference Equations: Linear Scalar Equations, Sufficient Conditions for Stability, Stability via Linearization, Global Stability of Nonlinear Equations, Applications, Flour Beetles, A Mosquito Model.

RECOMMENDED BOOKS:

1. Elaydi S., An Introduction to Difference Equations, 3rd ed., Springer-Verlag, New York, 2005.
2. Grove E.A., Ladas G., Periodicities in Nonlinear Difference Equations, Chapman and Hall/CRC Press, Boca Raton, 2004.
3. Sedaghat H, Nonlinear difference equations: Theory with applications to social science models, Kluwer Academic Publishers, Dordrecht, 2003.
4. Agarwal R.P., Difference Equations and Inequalities, Marcel Dekker, New York, 1992.
5. Edelstein-Keshet L., Mathematical Models in Biology, Random House, New York, 1988.
6. Brauer F., Castillo-Chavez C., Mathematical Models in Population Biology and Epidemiology, Springer-Verlag, New York, 2001.

MATH-710 **FUZZY ANALYSIS** **Credits: 3+0**

Elements of Set-Valued Analysis: Preliminaries, Hausdorff-Pompeiu Metric, Upper and Lower Semicontinuous Multifunctions, Hausdorff-Pompeiu Continuity, Closed Multifunctions, Continuous Selections, Measurable Multifunctions, Aumann integral, Hukuhara Derivative. Set Differential Equations: Preliminaries, Existence and Uniqueness, Successive Approximations. Fuzzy Sets: Fuzzy Sets, Level Sets, Special Types of Fuzzy Sets, Zadeh's Extension Principle, Fuzzy Functions, Sup-min Extension Principle, Interval Arithmetic, Fuzzy Numbers and Fuzzy Arithmetic. Fuzzy Metric Spaces: Fuzzy Metric Spaces, Inner Product, Support Function, Embendding Results, Continuous Fuzzy Functions, Measurable Fuzzy Functions, Integrable Fuzzy Functions, Differentiable Fuzzy Functions. Fuzzy Differential Equations: Preliminaries, Existence and Uniqueness, Fuzzy Functional Differential Equations, Fuzzy Differential Equations under Dissipative Conditions. Fuzzy Difference Equations: Basic Theory, Stability Results.

RECOMMENDED BOOKS:

1. Lakshmikantham V., Mohapatra R.N., Theory of Fuzzy Differential Equations and Inclusions, Taylor & Francis, New York, 2003.
2. Drainkov D., Hellendoorn H., Reinfrank M., An Introduction to Fuzzy Control, Springer, New York, 1996.
3. Aubin J.P., Cellina A., Differential Inclusions, Springer, Berlin, 1984.
4. Aubin J.P., Frankowska H., Set-Valued Analysis, Birkhäuser, Berlin, 1990.
5. Bector C.R., Chandra S., Fuzzy Mathematical Programming and Fuzzy Matrix Games, Springer-Verlag, Berlin Heidelberg, 2005.
6. Buckley J.J., Eslami E., Feuring T., Fuzzy Mathematics in Economics and Engineering, Physica-Verlag, Heidelberg 2002.
7. Buckley J.J., Eslami E., An Introduction to Fuzzy Logic and Fuzzy Sets, Physica-Verlag, Heidelberg, 2002.

MATH-712 ADVANCED INTEGRAL EQUATIONS Credits: 3+0

Introductory Concepts: Definitions, Classification of Linear Integral Equations, Fredholm Linear Integral Equations, Volterra Linear Integral Equations, Integro-Differential Equations, Singular Integral Equations, Volterra-Fredholm Integral Equations, Volterra-Fredholm Integro-Differential Equations, Solution of an Integral Equation, Converting Volterra Equation to an ODE, Differentiating Any Integral: Leibniz Rule, Converting IVP to Volterra Equation, Converting BVP to Fredholm Equation, Taylor Series, Infinite Geometric Series. Fredholm Integral Equations: Adomian Decomposition Method, Modified Decomposition Method, Noise Terms Phenomenon, Variational Iteration Method, Direct Computation Method, Successive Approximations Method, Method of Successive Substitutions, Comparison between Alternative Methods, Homogeneous Fredholm Integral Equations, Fredholm Integral Equations of the First Kind, Method of Regularization. Volterra Integral Equations: Adomian Decomposition Method, Modified Decomposition Method, Noise Terms Phenomenon, Variational Iteration Method, Series Solution Method, Converting Volterra Equation to IVP, Successive Approximations Method, Method of Successive Substitutions, Comparison between Alternative Methods, Volterra Integral Equations of the First Kind, Conversion of First Kind to Second Kind, Fredholm Integro-Differential Equations. Volterra Integro-Differential Equations, Volterra Integro-Differential Equations of the First Kind. Singular Integral Equations: The Weakly-Singular Fredholm Integral Equations. Nonlinear Fredholm Integral Equations: Nonlinear Fredholm Integral Equations of the Second Kind, Nonlinear Fredholm Integral Equations of the First Kind, Nonlinear Weakly-Singular Fredholm Integral Equations. Nonlinear Volterra Integral Equations. Applications of Integral Equations.

RECOMMENDED BOOKS:

1. Abdul-Majid W., A First Course in Integral Equations, World Scientific Publishing Co. Pte. Ltd., 2015.
2. Delves L.M., Walsh J., Numerical Solution of Integral Equations, Oxford University Press, London, 1974.
3. Estrada R., Kanwal R., Singular Integral Equations, Birkhauser, Berlin, 2000.
4. Jerri A., Introduction to Integral Equations with Applications, Wiley, New York, 1999.
5. Kanwal R.P., Linear Integral Equations, Birkhauser, Boston, 1997.
6. Porter D., Stirling D.S., Integral Equations: A Practical Treatment from Spectral Theory to Applications, Cambridge, 2004.

MATH-714 FRACTIONAL DIFFERENTIAL EQUATIONS Credits: 3+0

Preliminaries: Spaces of Integrable, Absolutely Continuous, and Continuous Functions, Generalized Functions, Fourier Transforms, Laplace and Mellin Transforms, The Gamma Function and Related Special Functions, Hypergeometric Functions, Bessel Functions, Classical Mittag-Leffler Functions, Generalized Mittag-Leffler Functions, Functions of the Mittag-Leffler Type, Wright Functions, Fixed Point Theorems. Fractional integrals and fractional derivatives: Riemann-Liouville Fractional Integrals and Fractional Derivatives, Liouville Fractional Integrals and Fractional Derivatives on the Half-Axis, Liouville Fractional Integrals and Fractional Derivatives on the Real Axis, Caputo Fractional Derivatives, Fractional Integrals and Fractional Derivatives of a Function with Respect to Another Function, Erdelyi-Kober Type Fractional Integrals and Fractional Derivatives, Hadamard Type Fractional Integrals and Fractional Derivatives, Grunwald-Letnikov Fractional Derivatives, Partial and Mixed Fractional Integrals and Fractional Derivatives, Riesz Fractional Integro-Differentiation. Ordinary fractional differential equations: Existence and uniqueness theorems. Methods for explicitly solving fractional differential equations.

RECOMMENDED BOOKS:

1. Podlubny I., Fractional Differential Equations, Academic Press, USA, 1999.
2. Kilbas A.A., Srivastava MH, Trujillo J.J., Theory & Applications of Fractional Differential Equations, Elsevier BV, Amsterdam, 2006.
3. Miller K.S., Ross B., An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley & Sons, Inc., 1993.
4. Zhou Y., Basic Theory of Fractional Differential Equations, World Scientific Publishing Co. Pte. Ltd., 2014.
5. Agrawal O.P., Machado J.T., Sabatier J., Fractional Derivatives and their Application: Nonlinear Dynamics, Springer-Verlag, Berlin, 2004.
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